

## COURBES DE CROISSANCE

$Q(t)$  : production cumulée

$Q_0 = Q(0)$  : valeur initiale de la production

$U$  : ressource ultime (finie !)

$a$  : taux d'accroissement de la production cumulée ( $0 \leq a \leq 1$  ou  $0 \leq a\% \leq 100$ )

$P = \frac{dQ}{dt}$  : production

$R = U - Q$  : réserve

Dans les exemples qui suivent :  $Q_0 = 10$  ;  $a = 0,1$  ;  $U = 100$

### Production constante, R/P décroissant

**Loi linéaire :**  $P = \frac{dQ}{dt} = a$

production cumulée

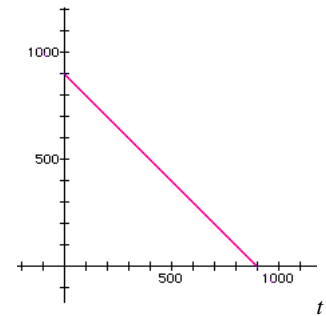
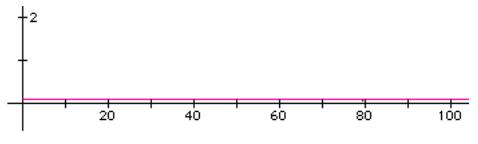
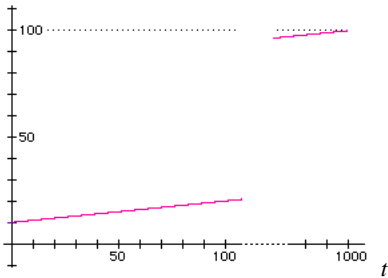
$$\Rightarrow Q = at + Q_0$$

production

$$P = \frac{dQ}{dt} = a$$

R/P

$$\frac{R}{P} = \frac{U - at - Q_0}{a}$$



### Production exponentielle, R/P décroissant

**Loi exponentielle :**  $P = \frac{dQ}{dt} = aQ$

production cumulée

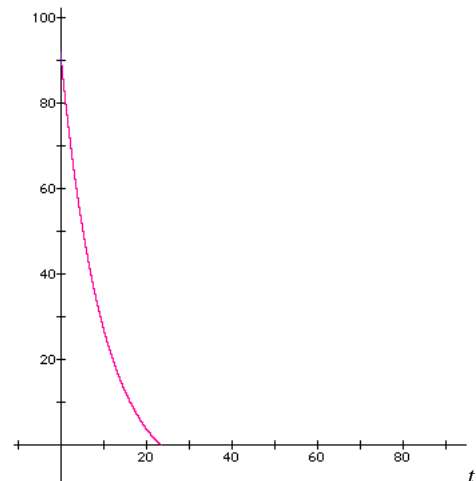
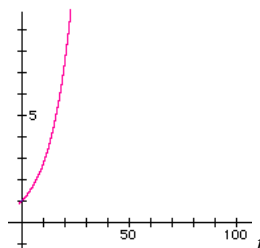
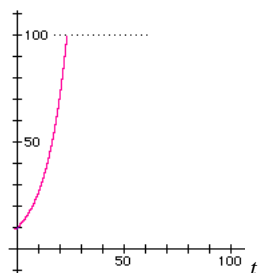
$$\Rightarrow Q = Q_0 e^{at}$$

production

$$P = \frac{dQ}{dt} = aQ_0 e^{at}$$

R/P

$$\frac{R}{P} = \frac{U - Q_0 e^{at}}{aQ_0 e^{at}}$$



### Production décroissante, R/P constant

**Loi du 1er ordre :**  $\frac{R}{P} = c \Rightarrow \frac{U - Q}{P} = c \Rightarrow c \frac{dQ}{dt} + Q = U$

production cumulée

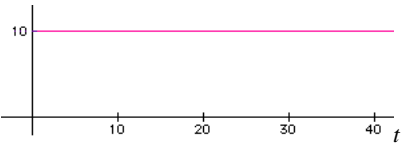
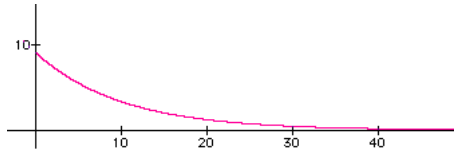
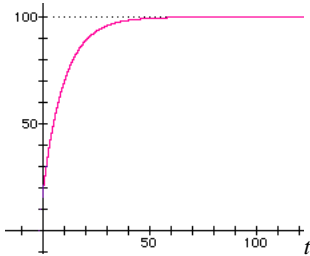
production

R/P

$$\Rightarrow Q = (Q_0 - U)e^{-t/c} + U$$

$$P = \frac{dQ}{dt} = c(U - Q_0)e^{-t/c}$$

$$\frac{R}{P} = c = \frac{1}{a} = 10 \text{ (exemple)}$$



### Production décroissante, R/P limité

**Loi logistique :**  $P = \frac{dQ}{dt} = aQ \left(1 - \frac{Q}{U}\right)$

production cumulée

production

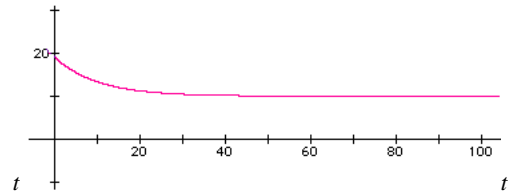
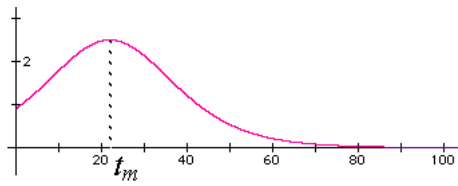
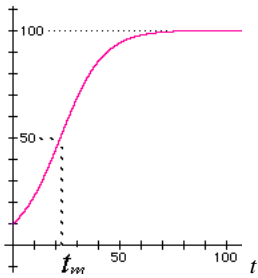
on pose :  $\alpha = \frac{U}{Q_0} - 1$

R/P

$$\Rightarrow Q = \frac{U}{1 + \alpha e^{-at}}$$

$$P = \frac{dQ}{dt} = \frac{U\alpha a e^{-at}}{(1 + \alpha e^{-at})^2}$$

$$\frac{R}{P} = \frac{U - \frac{U}{1 + \alpha e^{-at}}}{\frac{U\alpha a e^{-at}}{(1 + \alpha e^{-at})^2}}$$



$$Q(t_m) = \frac{U}{2}$$

$$t_m = \frac{\ln \alpha}{a}$$

$$\frac{R}{P} = \frac{1}{a} (1 + \alpha e^{-at}) \rightarrow \frac{1}{a}$$