COURBES DE CROISSANCE

$Q(t)$: *production cumulée*
$Q_0 = Q(0)$: valeur initiale de la production
U : ressource ultime (finie !)
a : taux d'accroissement de la production cumulée ($0 \le a \le 1$ ou $0 \le a_{\%} \le 100$)
$P = \dfrac{dQ}{dt}$: *production*
$R = U - Q$: *réserve*
Dans les exemples qui suivent : $Q_0 = 10$; $a = 0{,}1$; $U = 100$

Production constante, R/P décroissant

Loi linéaire : $P = \dfrac{dQ}{dt} = a$

production cumulée

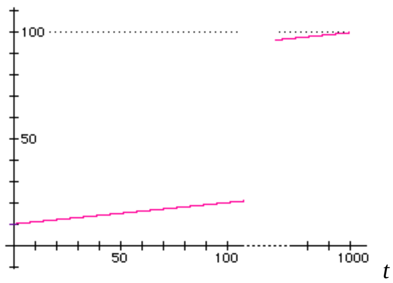
$\Rightarrow Q = at + Q_0$

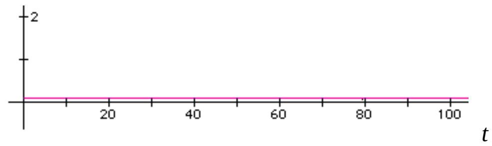
production

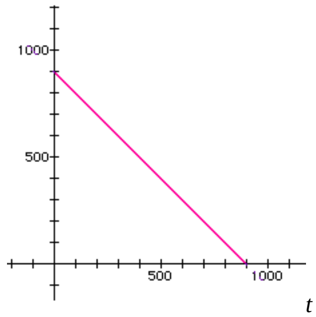
$P = \dfrac{dQ}{dt} = a$

R/P

$\dfrac{R}{P} = \dfrac{U - at - Q_0}{a}$







Production exponentielle, R/P décroissant

Loi exponentielle : $P = \dfrac{dQ}{dt} = aQ$

production cumulée

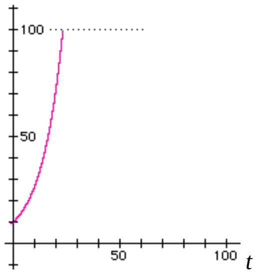
$\Rightarrow Q = Q_0 e^{at}$

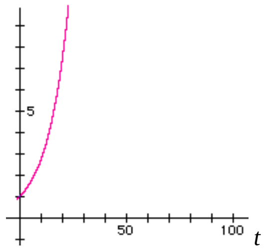
production

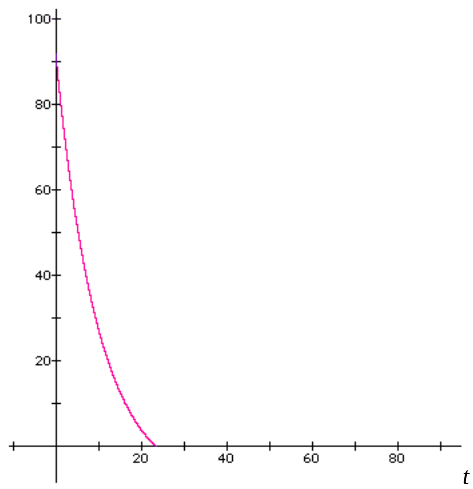
$P = \dfrac{dQ}{dt} = aQ_0 e^{at}$

R/P

$\dfrac{R}{P} = \dfrac{U - Q_0 e^{at}}{aQ_0 e^{at}}$







Production décroissante, R/P constant

Loi du 1er ordre : $\frac{R}{P} = c \Rightarrow \frac{U - Q}{P} = c \Rightarrow c\frac{dQ}{dt} + Q = U$

production cumulée

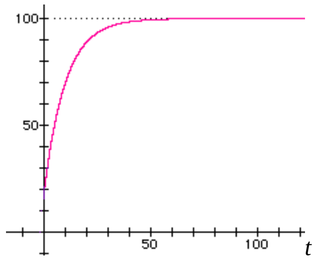
$\Rightarrow Q = (Q_0 - U)e^{-t/c} + U$

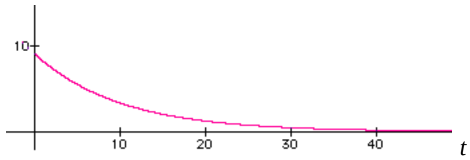
production

$P = \frac{dQ}{dt} = c(U - Q_0)e^{-t/c}$

R/P

$\frac{R}{P} = c = \frac{1}{a} = 10$ (exemple)





Production décroissante, R/P limité

Loi logistique : $P = \frac{dQ}{dt} = aQ\left(1 - \frac{Q}{U}\right)$

on pose : $\alpha = \frac{U}{Q_0} - 1$

production cumulée

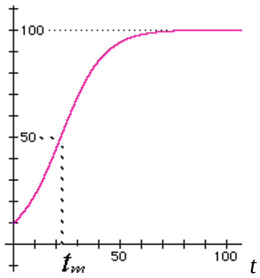
$\Rightarrow Q = \frac{U}{1 + \alpha e^{-at}}$

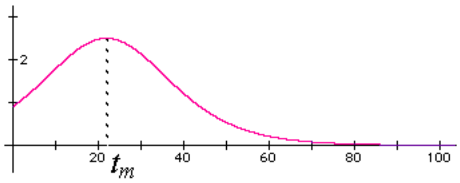
production

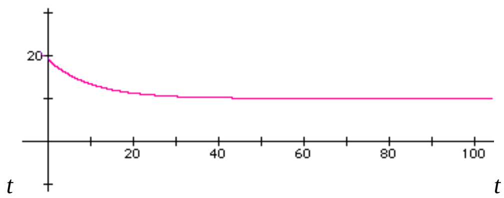
$P = \frac{dQ}{dt} = \frac{U\alpha a e^{-at}}{\left(1 + \alpha e^{-at}\right)^2}$

R/P

$\frac{R}{P} = \frac{U - \frac{U}{1 + \alpha e^{-at}}}{\frac{U\alpha a e^{-at}}{\left(1 + \alpha e^{-at}\right)^2}}$







$Q(t_m) = \frac{U}{2}$

$t_m = \frac{\ln \alpha}{a}$

$\frac{R}{P} = \frac{1}{a}\left(1 + \alpha e^{-at}\right) \rightarrow \frac{1}{a}$