COGNITIVE INFORMATION THEORY

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1- SIGNIFICANT INFORMATION

Significant information is a bipolar concept, informational and cognitive at once. So, to understand it, we have to study information as well as cognition structures (Pinson, 1994, 1995a, 1995b).

1.1- Information and identification contents

1.1.1- Ternary structure of information

An information content has only significance in relation to an other information content linked to a common code or context. A significant information is :

-*measurable by its proper content of information* (selective content, eigenvalue),

-*repaired by its explicit identification content* (metrical content, reference),

-*commensurable by its implicit identification content* (structural content, code, context, system of reference).

From receiver point of view, it is possible to calculate the different amounts of Shannon information contents being in a message. From transmitter point of view, it is possible to calculate the algorithmic information amount used to make the message. These calculus shows the information amounts lying in the message and lying in the code.

Necessity of an indentification content requires the existence of non-transmitted information, internal to the system. So there are two states for information :

-a *transmitted informational state,* or "kinetic" information.

-a *non-transmitted informational state* ("structural" or "eidetic" information), which is linked to the system.

1.2- Transmitted information

1.2.1- Information content

Transmitted information measure is evaluated from discernable states of a source. So extending concepts from discrete to continuous models encounters difficulties. It exists a smallest transmitted information quantity : YES/NO. Transmitted information is a quantized state based on a *distinction operator,* or indication (Spencer-Brown), which is depending on resolving power.

1.2.2- Identification contents

1.2.2.1- Explicit : it is possible to show with calculus of quantized transmitted information that resolving power is linked to sampling interval (Oswald). This is in time domain a demonstration of Nyquist 's sampling theorem usually established in frequency domain. Transmitted information is effectively transmitted, that is to say perceived, on condition that receiver quantizes to distinguish, and samples to quantize.

1.2.2.2- Implicit : transmitted – that is to say digital and sequential – information computing is a matter for some logical and mathematical rulers lying in Boole algebra, modulo 2 arithmetics, integers arithmetics. Order relationship in **N** allows univocal identification by counting transmitted samples.

On the other hand, distinction process being linked to receiver resolving power, if we accept information is the same whatever the size of physical phenomena which support it (like a message is the same in a low / loud voice), we must assume an *informational scale relativity principle* : signified invariance whatever signifier scale.

1.3- Non-transmitted information

1.3.1- Information content

The theory of calculability shows fundamental limits of computation. One of causes of

this limits lies in a basic theoreme of set theory (Wolper) : "the set of subsets P(**N**) of a countable set **N** is not countable". *We assume such results apply to communication scheme.* Messages transmitted are made of finite series of symbols which are quantized samples : the set of messages – i.e. the set of answers – is countable. But the set of knowledges, i.e. the set of languages (non-transmitted information linked to the receiver) characterizes the set of possible questions, i.e. the set of subsets of words (written by receiver in its alphabet), and so is not countable. There are (much) more questions than answers !

So, if the set of non-transmitted information, internal to the receiver, is not countable, it is a *continuum*. Each form is globally and simultaneously distributed to the wholeness of information set and vice versa. Well-knowed theorem of Taylor's serie of a continuous function shows that in every point of such a continuous object lies an information relating to its wholeness. As we will see, to be identifiable and meaningful, such a set must be consistent, that is ordered and coherent.

1.3.2- Identification contents

1.3.2.1- Explicit : this consistency results from an ordered *composition,* second fundamental operator with distinction. Receiver only understands that is linked to the wholeness of its informational universe. Some of mathematical bases of systems theory is a Laurent Schwartz theorem which shows that it is possible to describe any "black box" as a nucleus operating on input by volterration (Giré). This composition of functions, defined for the first time by Voltérra, leads to convolution. Eidetic information processing is a matter for rulers lying in convolution algebra or Lie algebra (correlation).

But in such a continuum, to define an information in relation to another, the meaning of a form is established in a consistent way by interfering with the internal informational set. Like hologram, a set of non-transmitted information has two properties :

- *simultaneous multiplicity* : each point of hologram contains an information on all of object , and each point of object is saved on all of hologram surface.
- *coherence* : each point is associated in a well-ordered way, by interfering, to all other points of set.

1.3.2.2- Implicit : But correlation algebra is a Lie algebra which is not commutative (Borsellino, Poggio). For instance is it possible to find this *non-commutability* of forms in chirality, which is an information typically impossible to transmit (except in some particular cases of weak radioactive disintegration) : this is an information where the significant variable is an oriented geometrical variable. In this way, information is conformation.

On the other hand, *theory of relativity* represents the internal point of view of universe system, and shows it spread out of all of its temporal and spatial extension. The "t" variable is an ordinary dimensional variable, that is to say symmetrical, commutative, implying virtual simultaneity property. The corollary of this is the impossibility to spot non-transmitted information in relation to temporal dimension.

2- COGNITIVE INFORMATION PROCESSING

2.1- Primary / discursive / multicrucial scheme

Experience shows there are several cognitive modalities as several sensory organs : *primary, discursive, multicrucial.* Tautologies production by primary modality is founded on a basic mind property : distinction ability. Multicruciality, as a "visual" modality, associating distinction and combination, classifies information with successive rearrangements (as Mendeleev did it), where eigenvalue information contents are associated in a common context from which they are spotted.

2.1.1- Incompleteness and Primary Logic

It is well-knowned that Continuum Hypothesis cannot be refuted (Gödel) netheir demonstrated (Cohen) in Zermelo-Fraenkel model. Intuition, in a way, is "larger" than rationality. Thought contains mathematics, the contrary is not true.

A rational assertion is distinguished from another one because contradictory assertions

eliminate each other : these mechanism lies on Primary Logic which allows us to distinguish A from ¬A. If discursive rationality lies on a falsifiability criterion, primary rationality rests on an apodictic criterion, which allows us to distinguish *a priori* the obvious from the absurd. A discursive theory is confirmed by facts, a statement of obvious confirms facts.

Distinguishing a rational assertion means, for the subject, to transmit this one to himself. This transmitted information is quantized, sampled and sequential (ordered belong time). Mathematics for instance are made of such series : Lövenheim-Skolem theorem seems to demonstrate it for set theory (there is a countable model for set theory – i.e. theory of countable and non-countable sets !). So *the power set of a rational assertions set is* $\aleph_0$.

But, facing a problem, we can imagine *a priori* an infinity of assertions. We can also imagine an infinity of axiomatics, that is to say subsets of assertions, which define new problems. Like the definition of a word, a problem or a question is linked to the wholeness of semantic universe of subject. So it is possible to see intuitive thought like the set of subsets of rational assertions set. *The power set of intuitive thought is the power set of Continuum,* $\aleph_1$.

2.1.2- Falsifiability and Multicruciality

Falsifiability expresses an other side of discursive rationality limits. An experience is said "crucial" when, acting as a criterion, it allows rejecting or confirming an hypothesis. In other words, among a continuous infinity ($\aleph_1$) of imaginable rational theories we make on the world (a theory can be rational and exact, as being false), a countable infinity ($\aleph_0$) among this theories will be at the same time a rational and true response to questions we ask us on the world. Filing and classifying experiences and distinguished pieces of information allow multiple cross-checking like cross-words, periodic table of elements, etc. Calculability theory show that any limit identification is the result of a classification (Delahaye). So there would be two criterions of "truth" : falsifiability which is a strong criterion, objective because independant of any system of reference, and multiple cross-checking which is a weak criterion, a criterion of scientificity making a system of reference, which does not say "world does like that or not", but only says "world does as if…"

2.2- Algebraic cognitive structures

The theory of *multicruciality* is the reason why we have to understand information processing as founded on a classification of some fundamental cognitive processes. We have seen some algebraic operators support this cognitive structures. It is possible to consider this structures as making a hierarchic diagram (fig. 1) where the same operator present at two different hierarchic levels owns a double meaning, as level in which it is : this is a possible definition of an "emergent behaviour". For each algebraic level, a distinction operator (marked Op) links up two associated composition operators which are a matter for a lower algebraic level and an upper respectively: $Op(a * b) = Op(a) \bullet Op(b)$ *versus* $Op^{-1}(a \bullet b) = Op^{-1}(a) * Op^{-1}(b)$.

"Op" is basically a "firmware" process which, acting on "hardware" operator marked * makes a "software" operator marked • at the next level (this process is reversible). Changing level, objects are changing of context, that is to say identification content.

3- SOME CONSEQUENCES OF COGNITIVE INFORMATION THEORY

3.1- Non-transmitted informational state properties

-It exists a parallel between boolean algebra and harmonic analysis : $\{\neg,\vee,\wedge,\emptyset,1\} <-> \{F,*,.,\delta,\mathbf{1}\}$. Note that, at this level, there are two distinct zeros : 0 which is nothingness, void, versus δ (Dirac function), the opposite of **1**.

-But F (Fourier Transform) is an operator which admit several fixed points : "comb" distribution (the indication of an ideally sampled infinite form is this form itself); gaussian function (noise makes no order, noise produces nothing but noise!). Now it was demonstrated by Lawvere that all classical negative results of limitation refer to non-satis-

faction of a fixed point property (Varela). So the problem of power set of non-transmitted information and intuitive thought (cf §2.1.1), which has no solution even if parallelism is put forward could find a solution with self-reference, developping the parallel between Spencer-Brown algebra and harmonic analysis.

-*Start* of a convolution is calculated summing product of *start* of form f by *start* of form g. But *start* of a correlation is calculated suming product of *start* of f by *end* of g : correlation calculation implies the whole implicit knowledge of the form. This is linked to virtual simultaneity property of non-transmitted information.

3.2-Transition between informational states

3.2.1- Perception operator

Physical harmonic analysis of forms follows sampling limitations : quantization, windowing, sampling, holding. So {F,*,.,δ,**1**} algebra is "digitalized", F becomes Digital Fourier Transform, whose calculus is performed by FFT "butterfly" algorithm (Fast Fourier Transform). The number of processing operations in this flow graph is $\Omega = (1/2)N.\log N$. This cellular networks topology is the same like a n-dimension hypercube where $n = \log N$. So we evaluate complexity of operating network which process multicrucial information by the ratio of optimal connection number on total connection number which is $\alpha = N(N-1)/2 \approx N^2/2$. So : $H = \Omega/\alpha = -(1/N)\log(1/N)$

This quantity is analogous to an information quantity: in this case, an information quantity is a quantity of dimensions (a "logon-content", number of independant variable features, said Gabor and MacKay).

3.2.2- Consequences

- Distinction operator (indication) is : f = F[g] i.e. f = ¬g . The information duality imply a fundamental indeterminacy principle, which ensues from indetermination relationship between a form and its spectrum (Vallée). From a cognitive point of view, it follows it is impossible to know with an arbitrarily sharp accuracy a form and its complement – this is the "specialist problem".

- F[F[f(x)]] = f(-x) : so, negation of negation of a form is not the form itself : there is a mirror effect, a"reflection": *«Va, je ne te hais point !»*

- $f \otimes g \neq g \otimes f$: perceived time irreversibility versus non-sequentiality internal information problem, that is to say irreversibility of time, is a consequence of the difference between ordered sequentiality of transmitted information and virtual simultaneity of non-transmitted information.

- At last, a strong objective approach of mind as a result of a measurement, so a sampled transmission of a nontransmitted informational state which is self-unmeasurable, is not possible. From an external observator point of view, it is impossible to access to internal information of a subject - except if the observator is the subject himself (self-reference of awareness).

3.3- "Super-Boole" computer

The above scheme of algebraic structures of information processing means the possibility of such a "Super-Boole" computer where intelligent calculus functions are implemented in the heart of boolean calculus : F as NOT operator, * as OR, . as AND, δ and **1** as boolean Ø and 1 respectively. By nature, it is impossible to comput like internal information processing, but informational states transition laws show how to simulate this one : we must translate non-transmitted operators by digitalization, applying independance scale principle. So the main operator, which is indication operator, ie Fourier Transform, becomes $F_1[f] = F[f]/\sup|F(f)|$. This principle of calculus does not modify the shape of form, only its amplitude. This allows extensions of boolean calculus, on any signals for example (like gaussians…), or fuzzy algorithmic, non-commutability, and so on. From separability of n-dimension Fourier Transform property, it is also possible to construct a "super-boolean" arithmetics acting on n-dimension numbers whose "bits" are themselves signals of N samples (harmonic binary numbers), and so on… This n-dimension, in-

dependant scale, harmonic analysis, where *n* could be as large than several hundred, could be a good neural network model of brain structure, and connections complexity could be explained by such principles.

3.3- Neuro-mathematics

Intuitive thought is for discursive rationality what real numbers are for integers. So mathematical modelization of cognition allows an asymptotic approach of thought, without never showing these one - basically contunuous and eidetic - but sampled and quantized yet. On the other hand mathematics, as pure product of mind, are an image of non-transmitted state information lying in mind. So it would be possible to construct a new approach of brain, whose mathematical properties are an indication of an internal frame.

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Figure 1